Optimization of electrostatic devices thanks to an hybrid BEM/FMM method

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Abstract — This article presents an optimization process working with a modeling method based on an hybrid Boundary Elements Method (BEM) and Fast Multipole Method (FMM). Actually, optimization step of designing an industrial device has taken great importance since relevant evolution of computer science. In that way, combining different modeling methods can bring important improvement to this step. After presenting the different modeling and optimization methods, the way they are made working together will be given and electrostatic application is presented.

I. INTRODUCTION

Designing an industrial device needs to predict the effects of its functioning. For an electrostatic device, it means being able to compute the electric fields or capacitance for example. To do so, different modeling methods exist, such as the most used Finite Element Method (FEM) and the Boundary Elements Method (BEM). Those ones both offer advantages and drawbacks. BEM induces less post processing noise due to an only surface meshing of the structure but is numerically limited from using full interaction matrixes. Anyway, its use is well adapted to structure optimization as only surface meshing has to be modified during each step. To overcome this limitation, combining this to a Fast Multipole Method (FMM) would bring great numerical improvement for using a finer meshing.

II. HYBRID BEM / FMM IN ELECTROSTATIC

Let first introduce the application area of the work which is the capacitance optimization of an electrostatic device. We define the capacitance C of a structure as follows:

$$C = Q/V = \sum_i^N Q_i/V_i \tag{1}$$

In this last equation, Q is the charge in Coulomb and V the potential in Volts, which repartition is the given information.

A. Use of a Boundary Element Method

In the right member of (1) is considered a surface meshing in N elements. On this meshing we consider a zero-order hypothesis where each quantity is constant by element. We also consider quantities at the geometric center of elements (point matching) which will define distances during integration. In most cases, structure geometry and voltage repartition are known, so only charges repartition has to be primarily computed. This can be done with a Boundary Element Method, building a N x N system:

$$[\mathbf{A}][\mathbf{Q}] = [\mathbf{V}] \tag{2}$$

The coefficients of matrix **A** can then be calculated thanks to the electrostatic equation:

$$A_{i,j} = \iint_{S_j} \frac{1}{4\pi.\varepsilon.r_{i,j}}.dS_j \tag{3}$$

In this last equation ε is the domain permittivity and r_{ij} the distance between the geometric center of element i and a varying point j of the source surface S_j. This coefficient can also be approximated by a sum based on placing Gauss points (correctly weighted) on S_j. After obtaining the full matrix A, the solution of (2) has several numerical difficulties when working with too many meshing elements. This problem can be overcome with a FMM.

B. Improvement thanks to a Fast Multipole Method

The FMM is a relative recent method that hierarchizes a meshing through a separation of proximate and far interactions. Different ways to build this meshing repartition have been developed but we will only focus on a fast adaptive multi-level one [1]. This aims to set a maximum number m of meshing elements in cubes. Practically, a first cube is built, containing all meshing elements (zero level). Then it is subdivided into eight new ones (level one) and only those who contain meshing elements are kept. If some cubes have more elements than m, they are subdivided once again until every lowest-level cubes have less or equal m elements (childless). After classifying all elements in this cubic hierarchy (Octree), a neighborhood study of each cube (of each level) is performed to separate most influencing cube towards others, creating 5 new groups [1].

At each cubic centre of childless element a multipole is computed from all charged elements contained inside. In spherical coordinates, this multipole creates an electrical potential V at a P point with the following equation:

$$V(P) = \sum_{n=0}^{p} \sum_{m=-n}^{n} M_n^m . Y_n^m(\theta,\varphi)/r^{n+1} \tag{4}$$

In this last equation (r, θ, φ) are the P coordinates, p the multipole order and the M_n^m the coefficient (which have to be computed one time for each childless cube):

$$M_n^m = \sum_{i=0}^{N} q_i . Y_n^{-m}(\alpha_i,\beta_i).\rho_i^n \tag{5}$$

where Y_n^m are Legendre polynomial coefficients, q_i the charge value of the i-element (for N contained in the cube) which has (ρ_i, α_i, β_i) spherical coordinates. After computing those coefficients, translation and conversion laws allow considering this multipole at a P point, which can be the center of another childless cube. In that way all elements far away from a childless cube can be approximated by locals written at its center, as shown in [1].

Remembering we want to get the capacitance of a structure from the knowledge of **V** repartition, we have to find **Q**, unknown vector of charge repartition on meshing elements. We have to write (4) for each meshing element, which give all far interactions. For close interactions (all other elements contained in the childless cube of P), a BEM is followed, creating a full square matrix for each childless cube. This can be summarized in the following system:

$$\mathbf{V} = \mathbf{P}.\mathbf{Q} + \mathbf{V}_{\mathbf{far}}(\mathbf{Q}) \quad (6)$$

In this last equation, **P** is a sparse matrix made of square sub matrixes and $\mathbf{V}_{\mathbf{far}}$ the potential created by far interactions thanks to (4).

C. Solving the system

We have to remark than system (6) is not directly solvable, as we do not have a direct interaction matrix like in (2). An iterative solver is needed, updating **Q** at each step. It is decided to choose the GMRES (General Minimum RESidual) one, well adapted to square symmetrical matrixes [2]. The main goal is not to solve a system but minimizing the following residual norm:

$$\mathbf{r} = \mathbf{P}.\mathbf{Q} + \left(\mathbf{V}_{\mathbf{far}}(\mathbf{Q}) - \mathbf{V}\right) \quad (7)$$

Different methods can be used to improve this method, such as the Arnoldi process and the Givens rotation [3]. A pre-conditioning of matrix P gives also better results by scaling **P** coefficients, thanks to the maximum and minimum eigenvalues. After N or less iterations, **Q** is obtained and the capacitance can be extracted.

III. USING HYBRID BEM/FMM IN OPTIMIZATION

Let focus on a particular electrostatic context as an applicative area. Main objective is to define some design criteria for minimizing an objective function based on capacitance computation:

$$OF(\mathbf{p}) = \left(C(\mathbf{p}) - C_m\right)^2 \quad (8)$$

In this last equation **p** is a vector of geometric parameters of the structure. To find the best parameters $\mathbf{p_f}$ from an initial $\mathbf{p_0}$, it is possible to use an iterative gradient based method, like the Steepest Descent [3]. This leads to define a normalized search direction $\mathbf{d_k}$ and a parameter α_k, depending $\mathbf{p_k}$. The process is the following:

1. Compute $\mathbf{d_k} = -\,\mathbf{grad}(OF(\mathbf{p_k})) / \|\mathbf{grad}(OF(\mathbf{p_k}))\|$
2. Find α_k that minimizes $OF(\mathbf{p_k} + \alpha_k.\mathbf{p_k})$
3. Update $\mathbf{p_{k+1}}$: $\mathbf{p_{k+1}} = \mathbf{p_k} + \alpha_k.\mathbf{p_k}$

The algorithm stops when the evaluation of $OF(\mathbf{p_{k+1}})$ is sufficiently proximate to $OF(\mathbf{p_k})$. To realize step 2, a quadratic line search can be done [3]. The main difficulty of this method is the evaluation of $\mathbf{grad}(OF(\mathbf{p_k}))$. Let focus on the example of 2 similar plates (-0.5V and 0.5V), forming a capacitor with 3 geometrical parameters h, s and e:

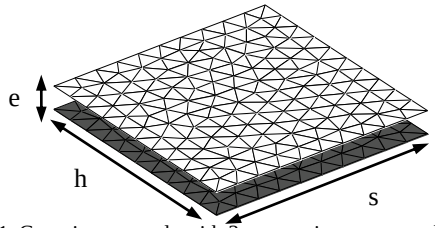

Fig. 1. Capacitor example with 3 geometric parameters h, s and e

Let first consider the BEM to model the physical behavior of this capacitor. Computing the gradient of the objective function leads to get the capacitance one and so the charges one. This can be done by using the adjoint state method [4], which needs a matrix relation between V and Q. This relation, written in (2), has to be derived by all parameters, that is to say building a matrix from the derivation of the integral terms in (3) by h, s and e. This gives the following result, after 11 iterations:

TABLE I THE OPTIMIZATION PROCESS WHEN CM = 6.6PF (WITH BEM)

Point	h (m)	s(m)	e (m)
Starting	0.2	0.2 m	0.05
Final	0.1027	0.1014 m	0.01987

Another difficulty stands then in introducing the FMM in this process as the interaction matrix A is no longer available. This can be done by summing for each geometric center the gradient of the close interaction matrix (as the previous method) and the FMM gradient (thanks to a derivation of (4) and (5) by h, s and e).

IV. CONCLUSION

This article has presented the use of an hybrid BEM/FMM method which allows computing quickly electrostatic quantities of a defined device. This method can be plugged into an optimization process, computing a chosen objective function. The optimization process chosen is deterministic: the Steepest Gradient. This iterative method requires the gradient of the modeling method which has to be carefully computed with the FMM.

V. REFERENCES

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